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CK-12 Texas Instruments Trigonometry

Student Edition

CK-12 Texas Instruments Trigonometry Student Edition

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CHAPTER 1 SE Introduction to Trigonometry - TI

Chapter Outline

1.1 Trigonometry TI Resources

Student Edition

Introduction

This FlexBook® resource contains Texas Instruments (TI) Resources for the TI-83, TI-83 Plus, TI-84, and TI-84 SE. All the activities in this resource supplement the lessons in the student edition. Teachers may need to download programs from www.timath.com that will implement or assist in the activities. All activities are listed in the same order as the Teacher's Edition. Each activity included is print-ready.

There are also corresponding links in CK-12 Trigonometry - Second Edition FlexBook® textbook.

- CK-12 Trigonometry - Second Edition: http://www.ck12.org/book/CK-12-Trigonometry-Second-Edition/

CHAPTER 2 SE Trigonometry and Right Angles - TI

Chapter Outline

The activities below are intended to supplement our Trigonometry FlexBook® textbooks.

- CK-12 Trigonometry - Second Edition: Chapter 1

2.1 Trigonometric Ratios

This activity is intended to supplement Trigonometry, Chapter 1, Lesson 3.

In this activity, you will explore:

- *Sine, cosine and tangents of angles*
- *Side length ratios of a right triangle*
- *Finding the missing side of a triangle when an angle and side are given.*

Before beginning this activity, make sure your calculator is in ***DEGREE*** *mode. Press the* **MODE** *button, arrow down two lines, arrow to the right to select* ***DEGREE****, and press* **ENTER**.

Problem 1 –Trigonometric Ratios

Start the **CabriJr** app by pressing the **APPS** button and choosing it from the menu.

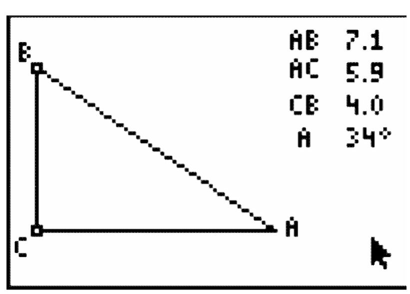
Select **Open** from the **F1: File** menu, and choose the file **TRIG**.

In right triangles, there is a relationship between the ratios of the side lengths and the trigonometric functions.

Find the ratios below using the values listed for the graph. Then find the values for the trig functions below using the value given for A on the graph.

$$\frac{BC}{AC} = \underline{\qquad}, \frac{AC}{AB} = \underline{\qquad}, \frac{BC}{AB} = \underline{\qquad}$$

$$\text{Sin } A = \underline{\qquad}, \text{Cos } A = \underline{\qquad}, \text{Tan } A = \underline{\qquad}$$



Repeat this for two more different triangles by moving point **B** to a different location (to grab a point, press the **ALPHA** button and use the arrow keys to move it.)

Triangle #2

$$\frac{BC}{AC} = \underline{\qquad}, \frac{AC}{AB} = \underline{\qquad}, \frac{BC}{AB} = \underline{\qquad}$$

$$\text{Sin } A = \_\_\_\_, \text{Cos } A = \_\_\_\_, \text{Tan } A = \_\_\_\_$$

Triangle #3

$$\frac{BC}{AC} = \_\_\_\_, \frac{AC}{AB} = \_\_\_\_, \frac{BC}{AB} = \_\_\_\_$$

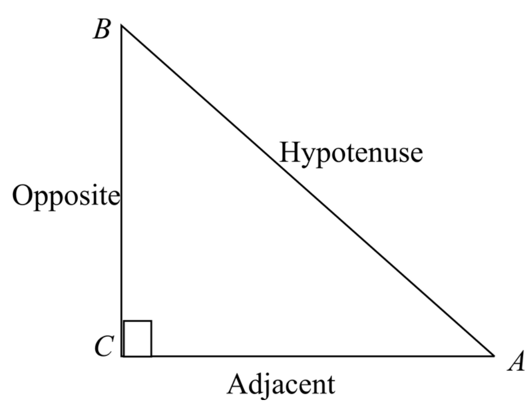
$$\text{Sin } A = \_\_\_\_, \text{Cos } A = \_\_\_\_, \text{Tan } A = \_\_\_\_$$

Based upon your answers hypothesize which ratio goes with each trigonometric function.

$$\text{Sin } A = \_\_\_\_\_\_\_\_ \quad \text{Cos } A = \_\_\_\_\_\_\_\_ \quad \text{Tan } A = \_\_\_\_\_\_\_\_$$

A good acronym to use to help remember these relationships is SOHCAHTOA

$$\sin\ A = \frac{\text{Opposite}}{\text{Hypotenuse}}$$
$$\cos\ A = \frac{\text{Adjacent}}{\text{Hypotenuse}}$$
$$\tan\ A = \frac{\text{Opposite}}{\text{Adjacent}}$$

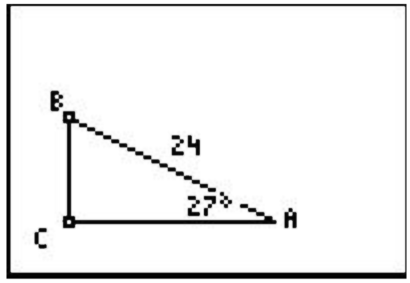


Problem 2 –Trigonometry, What Is It Good For?

One of the uses of trigonometry is finding missing side lengths of a triangle.

To find the length of side BC in the triangle to the right, write the sine relationship.

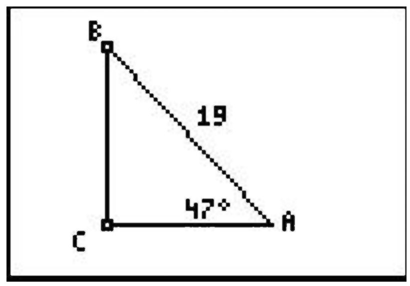
Find BC.



Now solve for *BC* and calculate using your handheld.

To find the length of side *AC* in the triangle to the right, write the cosine relationship.

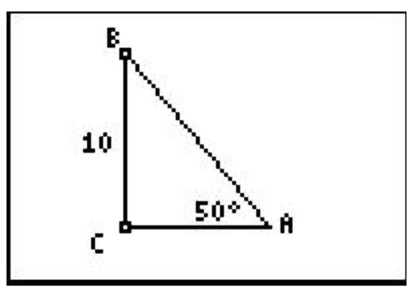
Find *AC*.



Now solve for *AC* and calculate using your handheld.

To find the length of side *AC* in the triangle to the right, write the tangent relationship.

Find *AC*.

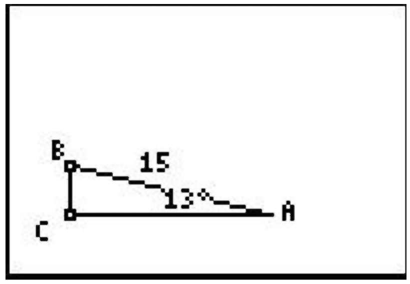


Now solve for *AC* and calculate using your handheld.

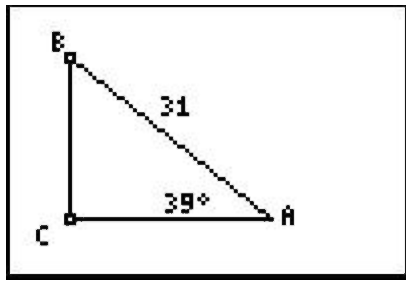
Student Exercises

Write the correct trigonometric function for each triangle below and solve for the missing side.

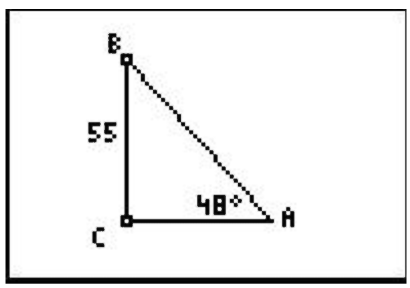
1. Find *AC*.



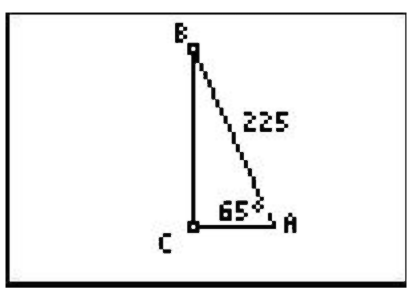
2. Find *BC*.



3. Find *AC*.



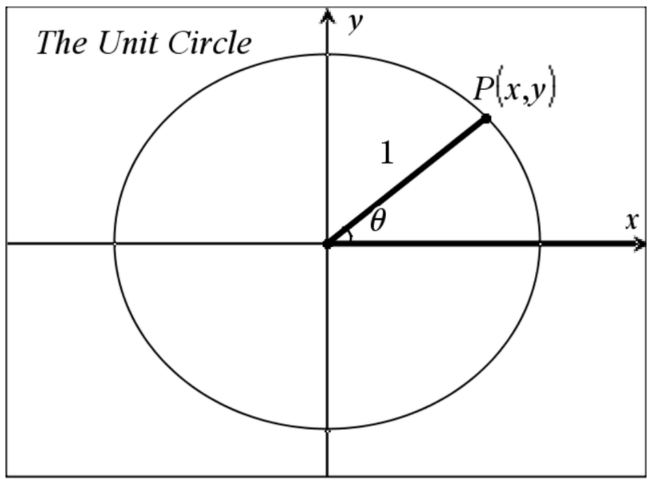
4. Find *AC*.



2.2 Round and Round She Goes

This activity is intended to supplement Trigonometry, Chapter 1, Lesson 7.

Problem 1 –Introduction to the Unit Circle



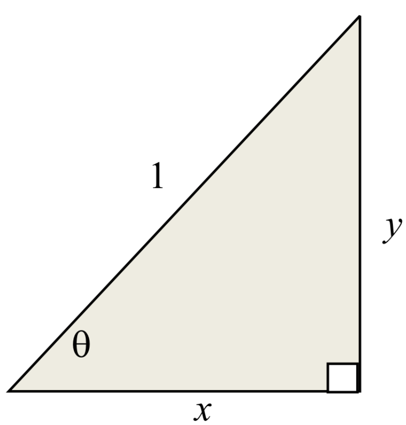
To the right, you will see a special circle known as the unit circle. It is centered at the origin and has a radius of one unit.

This circle is very important to the field of trigonometry. It is essential to develop an understanding of relationships between the angle theta, θ, and the coordinates of point P, a corresponding point on the circle.

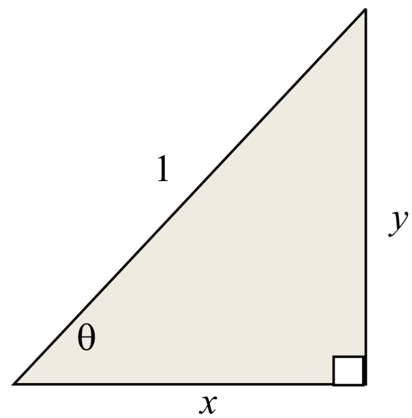
Note that the angle θ *is measured from the positive* $x-$*axis.*

Right triangle trigonometry and knowledge of special right triangles can be applied to understanding the relationship between θ and P. (Note that the hypotenuse of this triangle is 1 unit, corresponding to the radius of 1 unit on the unit circle.)

1. Using the right triangle diagram, write an equation for x in terms of θ.

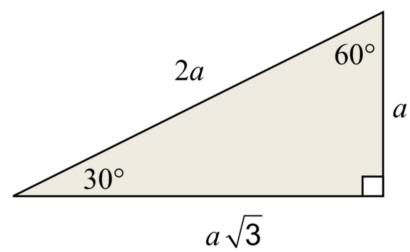


2. Using the right triangle diagram, write an equation for y in terms of θ.

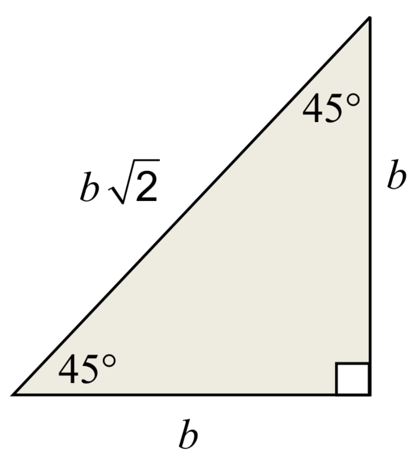


Using the answers to Exercises 1 and 2, the unit circle can be relabeled as shown to the right. Note that the $x-$value is $\cos(x)$ and the $y-$value is $\sin(x)$.

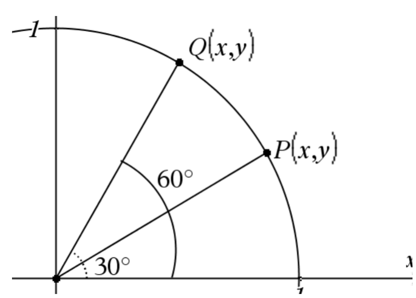
3. What is the value of a when the hypotenuse is 1 unit?



4. What is the value of b when the hypotenuse is 1 unit? Don't forget to rationalize the denominator!



5. Apply your knowledge of $30-60-90$ right triangles and identify the coordinates of point P.



6. Again, using your knowledge of $30-60-90$ right triangles, identify the coordinates of point Q.

7. The cosine of 30° is ________.

8. The sine of 30° is ________.

9. The cosine of 60° is ________.

10. The sine of 60° is ________.

Check your results to Exercises 7–8 using your graphing calculator as shown to the right.

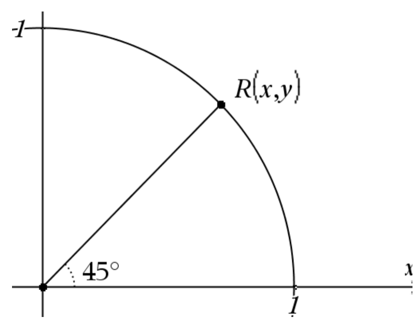
Note the $^\circ$ symbol can be found by pressing $2^{nd}+$ **[ANGLE]**; and then press **ENTER**.

11. Using your knowledge of $45-45-90$ right triangles, identify the coordinates of point R. _______

12. The cosine of 45° is ________.

13. The sine of 45° is ________.

Check your results to Exercises 11–13 using your graphing calculator.



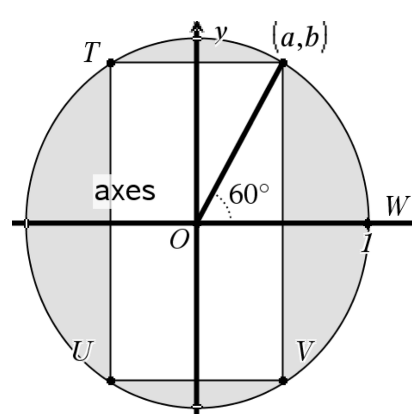
Problem 2 –Extending the Pattern

Identify the coordinates of the following points in terms of a and b.

14. T __________

15. U __________

16. V __________



Identify the measure of the following angles.

17. $m\angle WOT =$ ______

18. $m\angle WOU =$ ______

19. $m\angle WOV =$ ______

2.3 Trigonometric Patterns

This activity is intended to supplement Trigonometry, Chapter 1, Lesson 8.

The Unit Circle

Using the unit circle, the trig functions can be defined as follows:

$$\sin(\theta) = \frac{y}{h} \qquad \cos(\theta) = \frac{x}{h} \qquad \tan(\theta) = \frac{y}{x}$$

Using the *Cabri Jr.* application, drag the point on the circle in the first quadrant by pressing the **ALPHA** key and recording the value for $\sin\theta, \cos\theta$ and $\tan\theta$ using the displayed $x-$ and $y-$values.

Use the unit circle to complete the table.

TABLE 2.1:

θ	$\sin\theta$	$\cos\theta$	$\tan\theta$
30°			
45°			
60°			
90°			
120°			
135°			
150°			
180°			
210°			
225°			
240°			
270°			
300°			
315°			
330°			
360°			

Use the values in the table to answer the following questions.

1. For what values of θ is $\sin\theta$ positive?
2. For what values of θ is $\cos\theta$ negative?
3. For what values of θ is $\tan\theta$ positive? Negative? Why?
4. For what angle θ does $\cos\theta = \cos(30^\circ)$?
5. Name two other pairs of angles where the cosine of the angle is the same.
6. For what angle θ does $\tan\theta = \tan(45^\circ)$?
7. Name two other pairs of angles where the tangent of the angle is the same.
8. Record all the patterns you see with the sine function.

9. Are there any other patterns you see?

Extension –Patterns in Reciprocal Functions

Using the unit circle, the reciprocal trig functions can also be defined as

$$\csc(\theta) = \frac{h}{y} \qquad \sec(\theta) = \frac{h}{x} \qquad \cot(\theta) = \frac{x}{y}$$

Complete the following table by finding the reciprocals from the computed values on the earlier table.

TABLE 2.2:

θ	$\sec\theta$	$\csc\theta$	$\cot\theta$
30°			
45°			
60°			
90°			
120°			
135°			
150°			
180°			
210°			
225°			
240°			
270°			
300°			
315°			
330°			
360°			

Record any patterns you see.

CHAPTER 3 SE Circular Functions - TI

Chapter Outline

The activities below are intended to supplement our Trigonometry FlexBook® resources.

- CK-12 Trigonometry - Second Edition: Chapter 2

3.1 Find that Sine

This activity is intended to supplement Trigonometry, Chapter 2, Lesson 3.

Problem 1 –Temperature graphs

In this problem, you will graph and find a sinusoidal function.

The temperature in Kansas City fluctuates from cold in the winter to hot in the summer. The average, monthly temperature $(^{\circ}F)$ will be loaded into L_1 and L_2 when you run the program.

Press **PRGM** to access the Program menu.

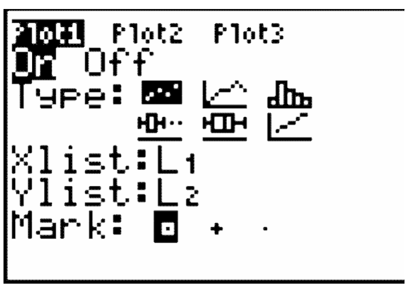
Choose the **KANSTEMP** program and press **ENTER.** This will load the six lists with the data for all three problems.

Press **STAT [EDIT]** to see the data in the lists. The number of the month is in L_1 and the temperature is in L_2.

Note: The data that appears in L_3 through L_6 will be used later in the activity.

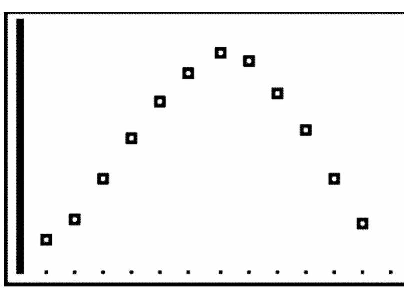
To graph the data, press 2^{nd} **[STAT PLOT]** and **ENTER** to access **Plot1**.

Make sure that the **Plot1** settings are the same as shown.



Press **ZOOM** and select **9:ZoomStat**.

You will get a graph similar to one to the right.



Find the sine equation that models the data. Press **STAT**, arrow over to **CALC**, and select **C:SinReg**. This brings the command to the home screen.

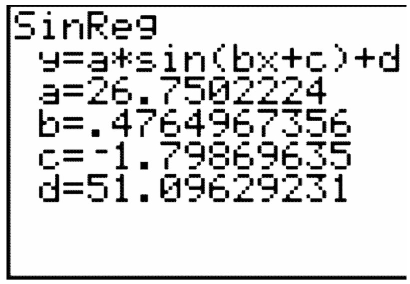
Follow the command by entering L_1, L_2, Y_1.

L_1 and L_2 can be entered by pressing the following keys: 2^{nd} $[L1]$ and 2^{nd} $[L2]$.

To enter Y_1, press **VARS** and arrow to select the **Y-VARS** menu. Then, press **ENTER** to select **1:Function...** and select $1 : Y_1$.

Press **ENTER** and the values for $a,\ b,\ c$, and d in the general formula $y = a\ ^*\ \sin(bx+c)+d$ will appear on the screen. The formula is now stored in Y_1.

- With two-decimal accuracy, record the sine equation:



Press **GRAPH** to see the sine regression with the data.

- How well does the sine equation model the data? Explain.

Clean the graph up by hiding the scatter plot.

To hide the scatter plot, press $Y =$ and arrow up to highlight **Plot1.** Press **ENTER.**

Press **GRAPH** to see the cleaned up graph.

Problem 2 –Hours of Sunlight

The amount of light a location on the Earth receives from the Sun changes each day depending upon the time of year and latitude of that location. The amount of daily sunshine Kansas City experiences has been recorded in the lists where the calendar day is in L_3, and the hours of sunlight is L_4.

Create the scatter plot, sine equation that models the data and "clean-up" the graph as outlined in Problem 1. To create the scatter plot, make sure to change the **XList** to L_3 and the **YList** to L_4.

In early cultures, certain days of the year had significant importance because of the planting cycle. These days were the winter and summer solstices, and the spring and fall equinoxes. The equinoxes are the days with equal amounts of light and dark. The summer solstice has the greatest amount of sunlight, while the winter solstice has the fewest amount of sunlight.

- With two-decimal accuracy, record the sine equation:
- How well does the sine equation model the data? Explain.

Find the four dates by tracing the equation and record the points below.

- $x1 =$ ________ $y1 =$ ________
- $x2 =$ ________ $y2 =$ ________
- $x3 =$ ________ $y3 =$ ________
- $x4 =$ ________ $y4 =$ ________

Problem 3: Tides

The Bay of Fundy has the highest tides in the world. If a tape measure were attached at the water line of a peer, and the water level height were recorded over a period of eighteen hours, data like that in L_5 and L_6 would be generated.

- With two-decimal accuracy, record the sine equation:
- How well does the sine equation model the data? Explain.
- Find the sinusoidal equation that models this data and predict the water level when the time is 49 hours after the readings were started. Since the sinusoidal regression equation will be stored in Y_1, use the function notation, $Y_1(49)$ on the home screen to predict the water level at 49 hours.

Additional Practice

The rabbit population in a field fluctuates with the seasons. In January, the cold weather and lack of food reduces the population to 500. In July, the population rises to its high of 800. This cycle repeats itself. Determine a model.

3.2 Vertical and Phase Shifts

This activity is intended to supplement Trigonometry, Chapter 2, Lesson 4.

Before beginning the activity, clear out any functions from the $Y =$ screen, turn off all Stat Plots, and make sure that the calculator is in Radian mode.

You are also going to utilize the **Transformation Graphing App**. To start this app, press **APPS** and select **Transform** from the list. Press **ENTER** twice to activate the application.

Problem 1 –Amplitude

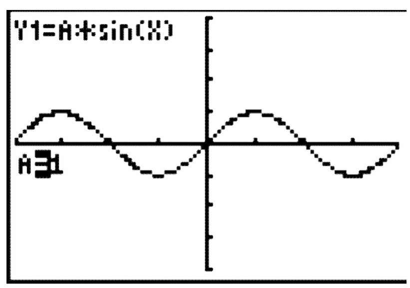
In this problem, you will explore the amplitude of a function of the form $f(x) = a\ \sin(x)$.

Press **PRGM** to access the Program menu and select the **PHSESHFT** program.

You will see the menu options to the right. Choose **1:Amplitude**.

The program will graph the parent function $y = \sin(x)$ and enter $Y_1 = a\ \sin(x)$ into the $Y =$ screen.

You will see $A = 1$ with the equals sign highlighted. Enter in values from -3.5 to 3.5 and press **ENTER** to see the effect the values have on the graph.



- Describe how the different values of a affect the shape of the graph.
- What happens if a is negative?
- Complete the following statement:

For $a \neq 0$, the graph of $Y_1 = a\ \sin(x)$ has an amplitude of ___________.

Problem 2 –Period

In this problem, you will explore the period of a function of the form $f(x) = \sin(bx)$.

Choose the **PHSESHFT** program again from the Program menu. Choose **2:Period**.

The program will graph the parent function $y = \sin(x)$ and enter $Y_1 = \sin(bx)$ into the o screen.

Enter the following values for B to see the effect the on the graph: 0.125, 0.25, 0.5, 0.75, 1, 2, 4, 8.

- Describe how the value of b affects the shape of the graph.

- What happens to the period when $0 < b < 1$?
- What happens to the period when $b > 1$?
- Complete the following statement:

For $b > 0$, the graph of $Y_1 = \sin(bx)$ has a period of _________.

Problem 3 –A simple phase shift

In this problem, you will explore the phase shift of a function of the form $f(x) = \sin(x+c)$

Choose the **PHSESHFT** program and select **3:Phase Shift**. The program will graph the parent function $y = \sin(x)$ and enter $Y_1 = \sin(x+c)$ into the $Y =$ screen.

Enter the following values for C to see the effect the on the graph: $-2\pi, -\frac{3\pi}{2}, -\pi, -\frac{\pi}{2}, -\frac{\pi}{4}, \frac{\pi}{4}, \frac{\pi}{2}, \pi, \frac{3\pi}{2}, 2\pi$

- Describe how the value of c affects the shape of the graph.

Problem 4 –Vertical shift

In this problem, you will review the vertical shift of a function of the form $f(x) = \sin(x) + d$.

Choose the **PHSESHFT** program and select **3:Vertical Shift**.

The program will graph the parent function $y = \sin(x)$ and enter $Y_1 = \sin(x) + d$ into the $Y =$ screen.

Enter the following values for D to see the effect the on the graph: -3 to 3 in 0.5 increments.

- Describe how the value of d affects the shape of the graph.
- Complete the following statement:

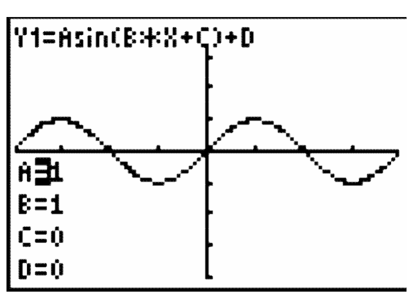
The graph of $Y_1 = a\ \sin(x) + d$ has a vertical shift of ___________.

Problem 5 –Combining transformations

In this problem, you will see which parameters impact the phase shift of the parent function, $y = \sin(x)$. You are to enter in various values for a, b, c, and d, and observe what happens. Try to write an equation that defines the phase shift in terms of the parameters that affect it.

Select the **TRIGCOMB** program from the Program menu. Choose **1:Phase Shift** from the menu options.

Enter values to change a, b, c, and d in the function $Y_1 = a\ \sin(bx+c) + d$.



- Which of the four parameters result in a phase shift of the graph?
- Complete the following statement:

For $a \neq 0$ and $b > 0$, the graph of $Y_1 = a\ \sin(b^{*}x + c) + d$ has a phase shift of ________

Problem 6 –Bringing it all together

- For functions of the form $f(x) = a\ \sin(bx + c) + d$, with $a \neq 0$ and $b > 0$, the graph has:
  - amplitude = _________
  - phase shift = _________
  - period = _________
  - vertical shift = _________

The same characteristics hold true for functions of the form $g(x) = a\ \cos(bx + c) + d$.

To verify this, press $Y =$ and arrow up to **Plot1** and press **ENTER** to turn off the stat plot. Then, change the sine equation to a cosine equation.

Press **GRAPH** and repeat Problems 1–4 with the cosine function.

Finally, you will apply what they have learned about vertical and phase shifts. You are given the equations and graphs of two sine functions and asked to find equations of cosine functions that coincide.

First, you first have to quit the **Transformation Graphing APP**. To do this, press the **APPS** and select **Transfrm** from the list. Choose **1:Uninstall**.

Select the **TRIGCOMB** program from the **Program** menu. Choose **2:Sine To Cosine** and select **1:EQN 1**.

You will see the graph of the following function:

$Y_1(x) = -1.5\ \sin\left(x + \frac{\pi}{4}\right) + 4$. Press $Y =$ and enter the cosine function into Y_2 that matches the sine function.

What is the equation that matches?

Clear out your function in Y_2.

Do the same process with the second equation. Choose the **TRIGCOMB** program and select **2:Sine To Cosine**. Choose **2:EQN 2** for the function: $Y_1 = 3\ \sin(2x) - 5$.

What is the equation that matches?

3.3 Getting Triggy With It

This activity is intended to supplement Trigonometry, Chapter 2, Lesson 6.

Problem 1 –A general trigonometric function

Using the *Transformation Graphing* app, press $Y =$ and enter the general sine function in Y_1,

$$Y_1 = A^* \sin(B^* X + C) + D.$$

Complete the table.

TABLE 3.1:

A	B	C	D	**zero1**	**zero2**	**min**	**max**
1	1	0	0				
4	$\frac{1}{2}$	3	1				

Problem 2 –The effect of the coefficients

Examining A

- Set $B = 1$ and $C = D = 0$ and change the value of A. Try 4 different values of A.

TABLE 3.2:

A	B	C	D	zero1	zero2	min	max
	1	0	0				
	1	0	0				
	1	0	0				
	1	0	0				

- How did the appearance of the graph change?
- Which graph features changed? Which did not change?
- Write equations to describe the relationship between A and the features that did change.
- When $B = 1$ and $C = D = 0$, ____________________.

The value of A is the **amplitude**. It is equal to half of the difference between its maximum and minimum values.

- Calculate the amplitude from the minimum and maximum values in the table above.
- Compare the results to the values of A. What do you notice?

Examining B

TABLE 3.3:

A	B	C	D	zero1	zero2	min	max
1		0	0				
1		0	0				
1		0	0				
1		0	0				

- Try 4 different values of B. How did the appearance of the graph change?
- Which graph features changed? Which did not change?
- Describe the relationship between B and the features that did change.

Examining C

TABLE 3.4:

A	B	C	D	zero1	zero2	min	max
1	1		0				
1	1		0				
1	1		0				
1	1		0				

- Try 4 different values of C. How did the appearance of the graph change?
- Which graph features changed? Which did not change?
- What is the effect of an increasing sequence of values for C on the graph?
- What is the effect of a decreasing sequence of values for C on the graph?

Examining D

TABLE 3.5:

A	B	C	D	zero1	zero2	min	max
1	1	0					
1	1	0					
1	1	0					
1	1	0					

- Try 4 different values of D. How did the appearance of the graph change?
- Try an increasing sequence of values for D such as 0, 1, 2, 3, 4 ... What is the effect on the graph?
- Try a decreasing sequence of values for D such as 0, -1, -2, -3, -4 ... What is the effect on the graph?
- Describe the effect of the value of D on the graph. How does changing D change the graph features?

Problem 3 –A closer look at amplitude, period, and frequency

In Y_1, enter the general cosine function, $A * \cos(B * X + C) + D$.

amplitude: half of the vertical distance from minimum value to maximum value

period: horizontal distance from one peak (maximum point) to the next

frequency: number of cycles per 2π interval

- Write a formula to find the frequency f given the period p.
- Use the formula to complete the table on the next page.

TABLE 3.6:

A	B	C	D	max point	min point	next max point	amplitude	period	frequency
1	1	0	0	(0, 1)	(3.14, -1)	(6.28, 1)	$\frac{1}{2} * (1 - (-1))$ 2	$6.28 - 0$ 6.28 2π	
	1	0	0						
	1	0	0						
1		0	0						
1		0	0						
1	1		0						
1	1		0						
1	1	0							
1	1	0							
1	1	0							

- Based on the results in the table, determine and record each relationship:
  - A and the amplitude
  - B and the frequency
  - B and the period

CHAPTER 4 SE Trigonometry Identities - TI

Chapter Outline

The activities below are intended to supplement our Trigonometry FlexBook® textbooks.

- CK-12 Trigonometry - Second Edition: Chapter 3

4.1 Trigonometric Identities

This activity is intended to supplement Trigonometry, Chapter 3, Lesson 1.

Problem 1 –Proving

Choose the **VERITRIG** program and select **PROVE ID 1**. Label the triangle with x, y, and θ in their respective places using the **Text** tool.

To prove: $\cos^2\theta + \sin^2\theta = 1$

1. Apply the Pythagorean Theorem to the right triangle:
2. Define the right triangle trig ratios for the triangle in terms of sine and cosine:
3. Substitute $x = \cos\theta$ and $y = \sin\theta$ into your equation from step 1:

Problem 2 - Proving

Select **PROVE ID 2**. Label the smaller triangle.

For the large triangle, the length of its base is 1. Let Y be the height and X be the hypotenuse. Label the large similar triangle accordingly.

Using our similar triangles, we can now set up proportional ratios. First, let's start by stating what we know: the ratio of the hypotenuse to the base is $\frac{1}{\cos\theta}$ (small $\triangle$) $= \frac{X}{1}$ (large $\triangle$)

And the ratio of the two sides of the small and large triangles: $\frac{Y(\text{large } \triangle)}{\sin\theta(\text{small } \triangle)} = \frac{1(\text{large } \triangle)}{\cos\theta(\text{small } \triangle)}$

- Cross multiply and state what X and Y are equal to:
- Now substitute these values into the Pythagorean Theorem:

Problem 3 –Numerical verification

Now that we have proved the two identities using our algebra skills, it is always nice to use the power of the calculator lists to numerically verify the two identities.

Select **NUM EXPLORE**. Trace to see the x values for cosine and the y values for sine for each angle measurement.

- As you move the cursor around the circle, state what patterns you notice between the $x-$and $y-$values.

These values are stored in the lists of the calculator with the angle measurements in $L1$, the cosine values in $L2$, and the sine values in $L3$. To numerically verify $\cos^2\theta + \sin^2\theta = 1$, you are going to use the lists of the calculator.

- What would you type into the top of a list (a formula) to numerically verify this Pythagorean Identity?

- Enter this in to the top of $L4$. State your results:

Again, to numerically verify $\sec^2\theta = 1+\tan^2\theta$, you are going to use the lists of the calculator.

- What would you type into the top of a list to numerically verify this Pythagorean Identity?
- Enter these in to the top of $L5$ and $L6$. State your results:
- Explain why you cannot verify this identity numerically using the lists of the calculator.
- What alternative method can you use to verify the identity?

Problem 4 –Verifying trig identities using graphing

Verify the identity $\sin^2 x = 1-\cos^2 x$ by graphing the two sides of the equation together.

You can check any identity with this method.

4.2 Trig Proofs

This activity is intended to supplement Trigonometry, Chapter 3, Lesson 2.

Problem 1 –Using the Calculator for verification

1. Prove: $(1+\cos x)(1-\cos x) = \sin^2 x$.

Verify the proof graphically. Enter the left side of the equation in Y_1 and the right side of the equation in Y_2.

Problem 2 - Confirm your findings

For questions 2 through 5, prove the equation given and then verify it graphically. For $\cot x$*, type* $\left(\frac{1}{\tan x}\right)$. *For* $\sec x$*, type* $\left(\frac{1}{\cos x}\right)$.

2. $\sin x \cdot \cot x \cdot \sec x = 1$

3. $\frac{\sec^2 x - 1}{\sec^2 x} = \sin^2 x$

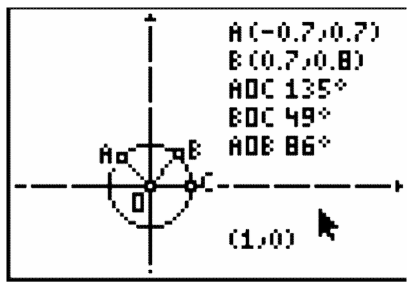
4. $\tan x + \cot x = \sec x(\csc x)$

5. $\frac{\sin^2 x - 49}{\sin^2 x + 14\sin x + 49} = \frac{\sin x - 7}{\sin x + 7}$

4.3 What's the Difference?

This activity is intended to supplement Trigonometry, Chapter 3, Lesson 5.

Problem 1 –Exploring the Angle Difference Formula for Cosine



Open the *Cabri Jr.* file called **UNITCIRC**. Observe a unit circle with points A, B, and C on the circle. Point O is the origin and center of the circle. The central angle $\angle AOB$ represents the difference between $\angle AOC$ and $\angle BOC$.

Move points A and B to see the changes to the on-screen measurements. To select an object, press the **ALPHA** key.

1. The $x-$coordinate of every ordered pair of a point on the unit circle represents the ________________________ of the corresponding angle.

2. The $y-$coordinate of every ordered pair of a point on the unit circle represents the ________________________ of the corresponding angle.

Use the file **UNITCIRC** to answer the following questions.

3. What is the sine of $\angle AOC$ when its measure is about 100°?

4. What is the cosine of $\angle AOC$ when its measure is about 100°?

5. What is the sine of $\angle BOC$ when its measure is about 20°?

6. What is the cosine of $\angle BOC$ when its measure is about 20°?

7. What is the sine of $\angle AOC - \angle BOC$ when $m\angle AOC = 100^\circ$ and $m\angle BOC = 20^\circ$? Use the Cabri Jr. file to obtain your solution.

8. What is the cosine of $\angle AOC - \angle BOC$ when $m\angle AOC = 100^\circ$ and $m\angle BOC = 20^\circ$? Use the Cabri Jr. file to obtain your solution.

9. Do you think the relationship between the values of sine and cosine for $\angle AOC - \angle BOC$ is quickly and easily obtained from the two individual angles as shown on the opening diagram for this activity? Explain you answer.

Problem 2 –Applying the Angle Difference Formula

The opening diagram for this activity is commonly used in the derivation of the angle difference formula for cosine.

$$\cos(A-B) = \cos A \cdot \cos B + \sin A \cdot \sin B$$

This formula is useful in finding exact values for the cosine of angles other than those you may already know from the unit circle.

For each exercise below, use your graphing calculator first with and without the formula. Then use the **UNITCIRC** circle graph.

10. Find the value of $\cos 15°$ by finding $\cos(60° - 45°)$.

11. Find the value of $\cos 75°$ by finding $\cos(120° - 45°)$.

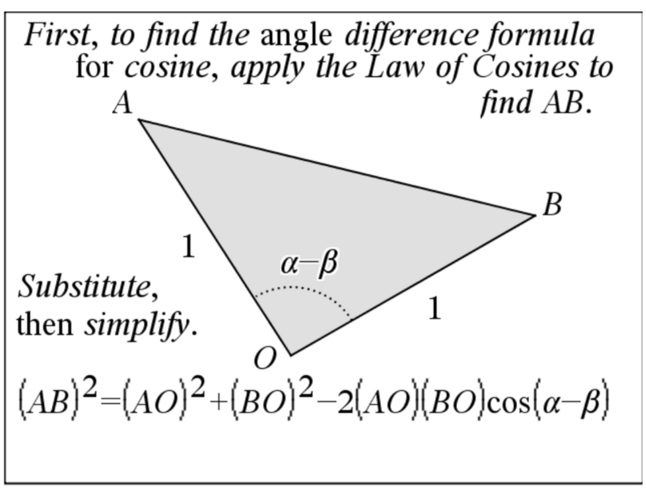
12. Find the value of $\cos 105°$ by finding $\cos(? - ?)$. You choose the angles! Choose values that you recall from the unit circle.

Extension 1 –Derivation of the Angle Difference Formula for Cosine

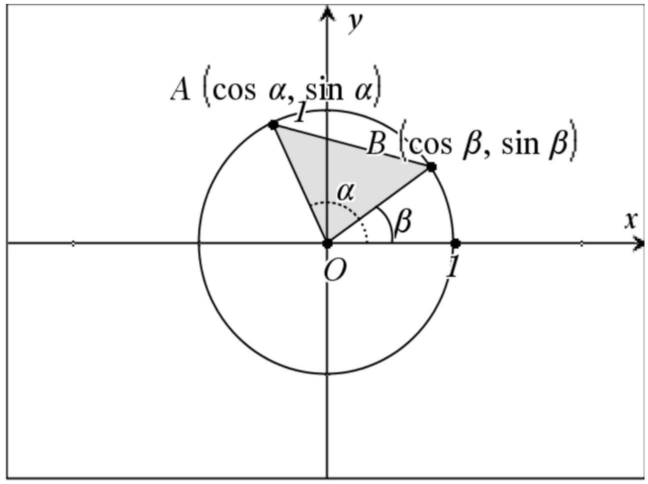
The *Cabri Jr.* file that has been used for this activity will be used in the derivation of the angle difference formula for cosine. As you look at the sketches below, find the angle represented by $\alpha - \beta$. Points A and B may be moved to change the measures of angles in the diagram.

The angle difference formula for cosine will be derived using the diagrams below.

13. Apply the Law of Cosines to the figure to the right to find an equation representing AB^2.



14. Apply the distance formula to the figure to the right to find an equation representing AB^2.



15. Combine the two equations obtained in Exercises 13 and 14 by setting them equal to each other. Solve for $\cos(\alpha - \beta)$. Test your resulting equation by entering values of your choice in **UNITCIRC**. Does your result agree with the directly calculated angle difference value? If not, check for algebraic and calculator entry errors.

Extension 2 –Derivation of the Angle Sum Formula for Cosine

16. Now substitute $-\beta$ in place of β into the angle difference formula for cosine and simplify the resulting equation. Test your resulting equation by entering values of your choice on page 3.3. Does your result agree with the provided angle sum value? If not, check for algebraic and calculator entry errors.

CHAPTER 5 SE Inverse Functions and Trigonometric Equations - TI

Chapter Outline

This activity is intended to supplement our Trigonometry FlexBook® textbooks.

- CK-12 Trigonometry - Second Edition: Chapter 4

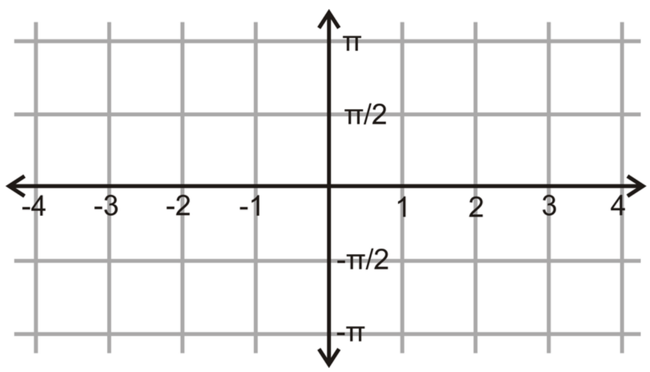
5.1 What's your Inverse?

This activity is intended to supplement Trigonometry, Chapter 4, Lesson 3.

Problem 1

Press $Y =$ and graph $Y1 = \sin^{-1} x$. Press **MODE** and make sure **Radian** is highlighted. Press **GRAPH**.

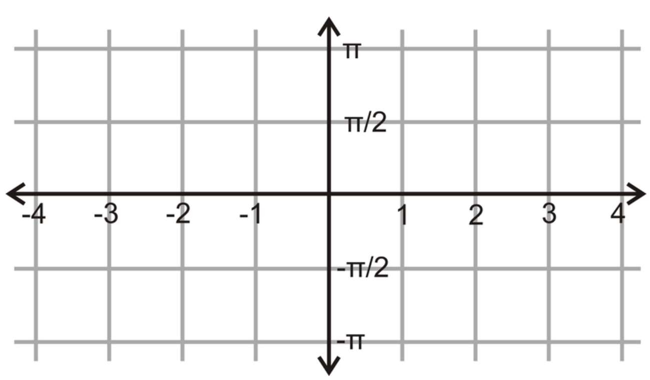
- Press **ZOOM**, **7:ZTrig**. Graph and determine the domain and range of the function.
- Why is there a restricted domain on this function?



Problem 2

Press $Y =$ and graph $Y1 = \cos^{-1} x$. Press **GRAPH**.

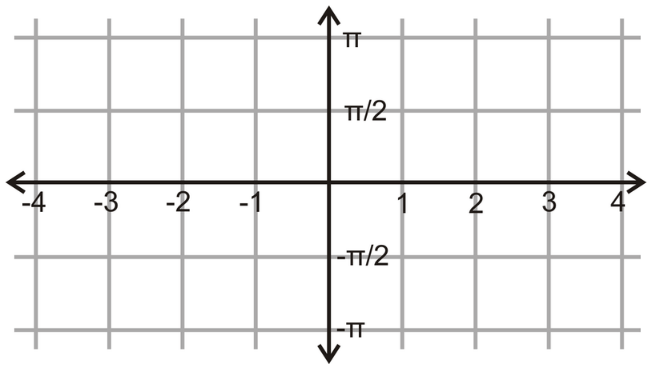
- Graph and determine the domain and range of the function.



Problem 3

Press $Y =$ and graph $Y1 = \tan^{-1} x$. Press *'GRAPH*.

- Graph and determine the domain and range of the function.

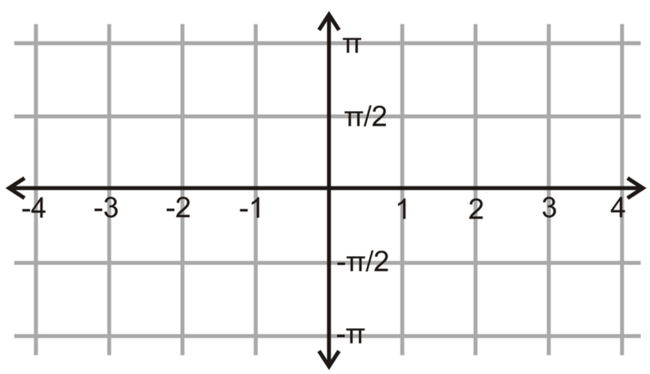


For secant, cosecant and cotangent, it is a little more difficult to plug into $Y =$.

Problem 4

Prove $\cos^{-1} x = \sec^{-1}\left(\frac{1}{x}\right)$. This will be how you graph $y = \sec^{-1} x$ in the graphing calculator.

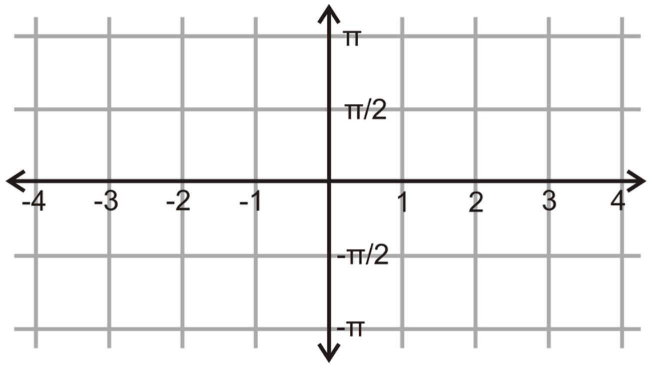
- Graph your results from above in $Y =$. Find the domain and range of the function.



Problem 5

Prove $\sin^{-1} x = \csc^{-1}\left(\frac{1}{x}\right)$. This will be how you graph $y = \csc^{-1} x$ in the graphing calculator.

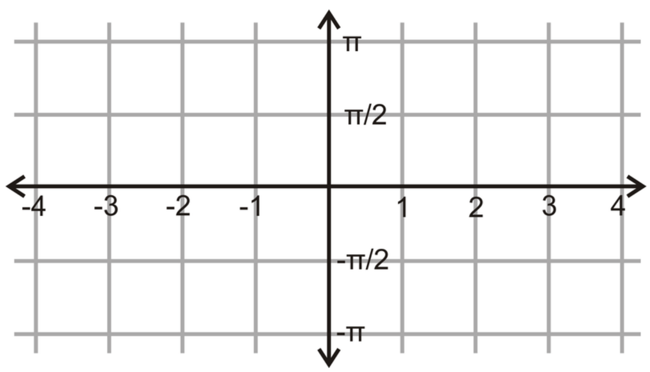
- Graph your results from above in $Y =$. Find the domain and range of the function.



Problem 6

Tangent and cotangent have a slightly different relationship. Recall that the graph of cotangent differs from tangent by a reflection over the $y-$axis and a shift of $\frac{\pi}{2}$. As an equation, it would be $\cot x = -\tan\left(x - \frac{\pi}{2}\right)$. Take the inverse of $y = -\tan\left(x - \frac{\pi}{2}\right)$.

- Graph your results from above in $Y =$. Find the domain and range of the function.



CHAPTER 6 SE Triangles and Vectors - TI

Chapter Outline

The activities below are intended to supplement our Trigonometry FlexBook® textbooks.

- CK-12 Trigonometry - Second Edition: Chapter 5

6.1 Analyzing Heron's Formula

This activity is intended to supplement Trigonometry, Chapter 5, Lesson 2.

Problem 1 - Consider a 3, 4, 5 right triangle.

- Draw the triangle.
- Find the area using $A = \frac{1}{2}bh$.
- Consider Heron's Formula, $A = \sqrt{s(s-a)(s-b)(s-c)}$

In $Y =$, plug into $Y1 = \sqrt{x(x-3)(x-4)(x-5)}$. Zoom in by changing the window. Press **WINDOW** and change the parameters to the right.

Press **GRAPH.**

WINDOW

$Xmin = -1$

$Xmax = 8$

$Xscl = 1$

$Ymin = -1$

$Ymax = 10$

$Yscl = 1$

$Xres = 1$

- Describe the graph. Does it have any x or y intercepts?

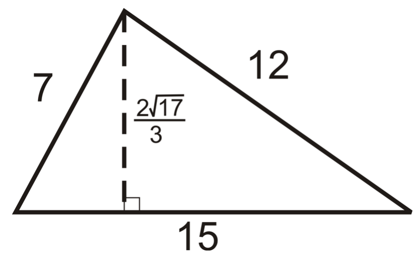
- In $Y2$, type $Y2 = \frac{1}{2} \cdot 3 \cdot 4$ or 6, the area of this triangle. Press **GRAPH.** Do the two functions intersect? If so, write the point(s) below.

- $Y1$ is Heron's formula with $a = 3$, $b = 4$, and $c = 5$ and $s = x$. What do the point(s) above tell us about this specific Heron's formula? What do (x, y) represent?

Problem 2

Repeat the steps from Problem 1 with the triangle below.

Are your findings the same?


7
12
$\frac{2\sqrt{17}}{3}$
15

6.2 Sine. It's the Law.

This activity is intended to supplement Trigonometry, Chapter 5, Lesson 3.

Problem 1 –Law of Sines

Open the *Cabri Jr.* application by pressing **APPS** and selecting **CabriJr**. Open the file **LAW1** by pressing $Y =$, selecting **Open...**, and selecting the file. You are given $\triangle ABC$ with the measure of all angles and sides calculated.

1. Grab and drag point B (use the **ALPHA** button to grab the point), and record the values of a, b, c, $\angle A$, $\angle B$, and $\angle C$. Repeat this three more times.

TABLE 6.1:

Position	a	b	c	A	B	C
1						
2						
3						
4						

2. On the calculator home screen calculate $\sin(A)$, $\sin(B)$, and $\sin(C)$. Then, calculate the following ratios: $\frac{\sin(A)}{a}$, $\frac{\sin(B)}{b}$, and $\frac{\sin(C)}{c}$.

TABLE 6.2:

Position	$\sin(A)$	$\sin(B)$	$\sin(C)$	$\frac{\sin(A)}{a}$	$\frac{\sin(B)}{b}$	$\frac{\sin(C)}{c}$
1						
2						
3						
4						

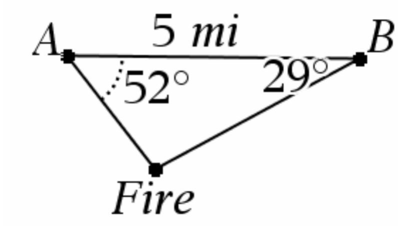
3. What do you notice about the last three columns of the table in Question 2?

4. Make a conjecture relating $\frac{\sin A}{a}$, $\frac{\sin B}{b}$, and $\frac{\sin C}{c}$.

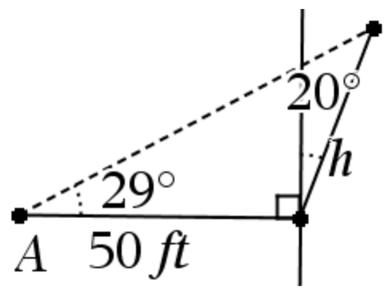
Problem 2 –Application of the Law of Sines

5. State the Law of Sines.

6. The distance between two fire towers is 5 miles. The observer in tower A spots a fire 52° SE and the observer in tower B spots the same fire 29° SW. Find the distance of the fire from each tower.



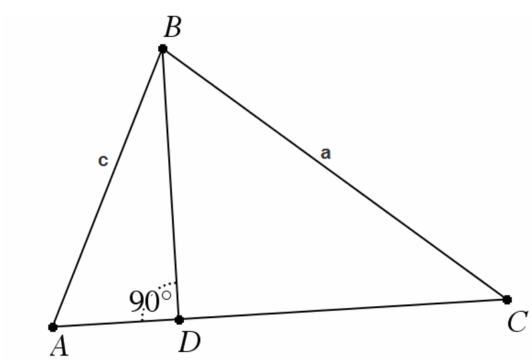
7. A tree leans 20° from vertical and at a point 50 ft. from the tree the angle of elevation to the top of the tree it 29°. Find the height, h, of the tree.



8. A boat is spotted by lighthouse A at 25° NE and spotted by lighthouse B at 50° NW. The lighthouses are 10 miles apart. What is the distance from the boat to each lighthouse?

Extension –Proof of the Law of Sines

We will now prove the Law of Sines. We will prove that $\frac{\sin(A)}{a} = \frac{\sin(C)}{c}$. You can use similar methods to show that $\frac{\sin(A)}{a} = \frac{\sin(B)}{b}$ and $\frac{\sin(B)}{b} = \frac{\sin(C)}{c}$. You are given $\triangle ABC$, altitude BD, and sides a and c.



9. Using right triangular trigonometry, what is the sine ratio for $\angle A$?

10. Using right triangular trigonometry, what is the sine ratio for $\angle C$?

11. What side is common to the sine of A and the sine of C? Solve for this common side in the ratio for sine of A and sine of C.

12. Since the side from Exercise 13 is common to both equations we can set them equal to each other. Set your two equations equal and try to show that $\frac{\sin(A)}{a} = \frac{\sin(C)}{c}$.

CHAPTER 7 SE Polar Equations and Complex Numbers - TI

Chapter Outline

This activity is intended to supplement our Trigonometry FlexBook® textbooks.

- CK-12 Trigonometry - Second Edition: Chapter 6

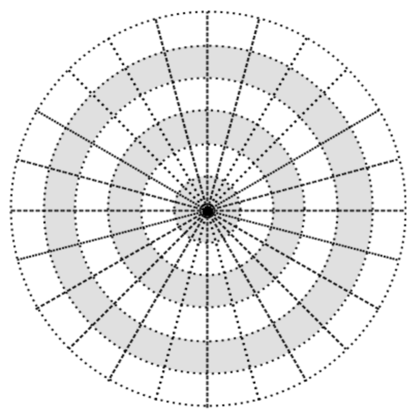
7.1 Polar Necessities

This activity is intended to supplement Trigonometry, Chapter 6, Lesson 4.

Plotting Coordinates & Exploring Polar Graphs

The coordinates of a polar curve are given as $(\theta,\ r)$.

1. Plot and label the following points on the graph below: $A(15^\circ,\ 4)$, $B(270^\circ,\ 5)$, $C\left(\frac{\pi}{6},3\right)$ and $D\left(\frac{3\pi}{2},6\right)$.



2. If $r(\theta) = \cos(\theta)$, what is $r\left(\frac{\pi}{3}\right)$?

3. Graph $r(\theta) = 2-2\cos(\theta)$. What is the shape of the graph?

4. Using your graphing calculator, explore polar graphs by changing the equation from #3. Try to generate the graphs listed below. Which of the graphs were you able to make? Write the equation next to the graph shape.

- circle
- rose with even number of petals
- rose with odd number of petals
- limaçon with an inner loop